# Perfection for Pomonoids

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A *pomonoid* is a monoid $S$ partially ordered by $\leq$ such that $\leq$ is compatible with the monoid operation. A lot of written work has appeared on partially ordered algebraic systems in the literature to date, e.g. on ordered groups and ordered rings. In 1980 **Fakhuruddin** was the first to start investigating the properties of ordered sets acted on by ordered monoids, called $S$-*posets*. This area lay fallow until this decade; a good collection of papers have appeared in the last few years.

A ring $R$ is *left perfect* if every left $R$-module has a projective cover; similar definitions exist for monoids and pomonoids. Investigation of left perfect rings was started by Bass [1] in 1960, where he proved that these are precisely those rings satisfying Condition $(M_R)$, i.e the descending chain condition on principal right ideals. In 1970 Isbell [3] was the first who took the initiative to investigate left perfect monoids. The result of Isbell together with those given by Fountain [2] showed that a monoid is left perfect if and only if it satisfies Condition $(M_R)$ and in addition a Condition " ascending chain condition on cyclic $S$-subacts " namely $(A)$.

In [4] we started the study of left poperfect pomonoids. We have shown that a pomonoid is left poperfect if and only if it satisfies two Conditions, $(M_R)$ and an ordered version of Condition $(A)$, denoted by $(A)^O$. Condition $(A)$ and $(A)^O$ are related with the direct limits of copies of $S$. A careful study of direct limits enables us to show that $(A)$, and $(A^O)$ are equivalent. Hence a pomonoid $S$ is left perfect if and only if it is left poperfect. We also show that a pomonoid is left perfect if and only if every strongly flat left $S$-poset is projective, a result which is again analogous to that in the ring and monoid situations. At some places we follow the arguments as given in monoid case but care is needed to deal with the order involved while at some places we follow new techniques.

## References


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